

Differential cryptanalysis

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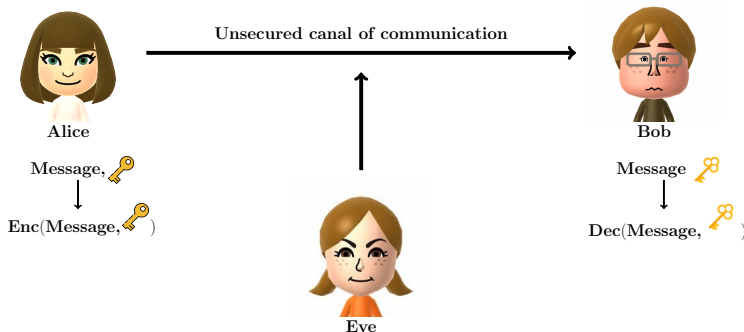



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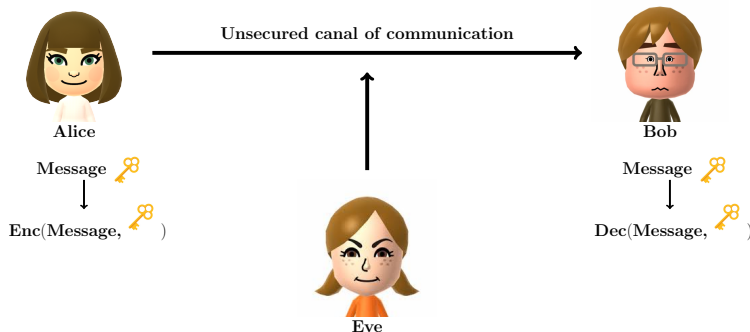
- ① Symmetric cryptography
- ② Differential Cryptanalysis
- ③ Key recovery attack


Symmetric encryption



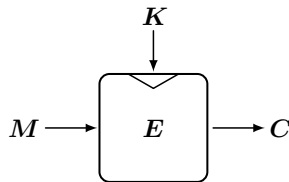
- **Goal:** Ensure that only the authorized entities has access to the message.
- Secret key  shared beforehand.

Symmetric encryption



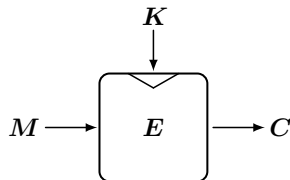
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Block ciphers



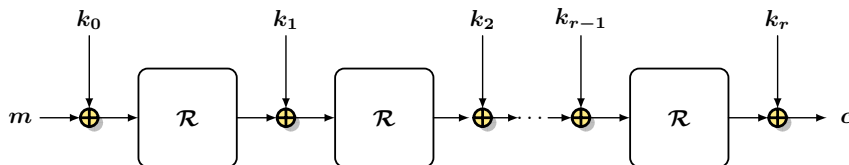
- E is a function $\mathbb{F}_2^n \times \mathbb{F}_2^k \rightarrow \mathbb{F}_2^n$.
- Block of size n of 64 or 128 bits in general.
- Key of size k of 128 or 256 bits in general.

Block ciphers

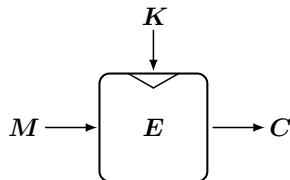


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Usual structure of a block cipher:

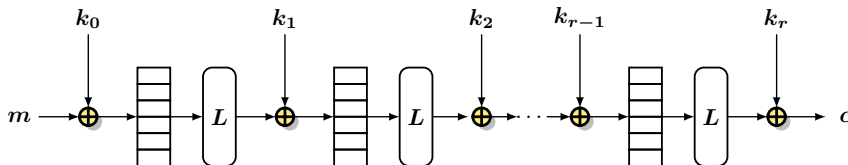


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The security is defined by the best **generic attack**.

↪ Exhaustive search of the secret key.

- Best attack against an ideal cipher.
- Test all the possible key with a known pair (M, C) .
- Cost 2^k encryption.
- The size k of the key need to be big enough.

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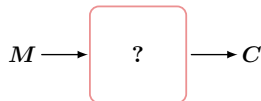
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- Best attack against an ideal cipher.
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⇒ Cryptanalysis is needed to trust the security of the ciphers.

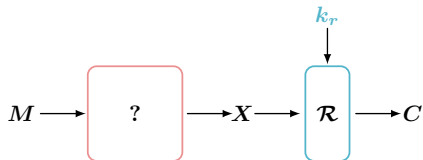
What is an attack?

① Distinguisher



- Find property to distinguish a block cipher from a random permutation with a high probability.
- Number of queries \rightsquigarrow complexity of the distinguisher.

② Last round attack



- Transform the distinguisher into key-recovery attack.
- For each guess of $k_r \rightarrow$ check if (M, X) verify the distinguisher properly.
- Can add more than one round.

How to measure the efficiency of an attack?

Three complexities to measure an attack:

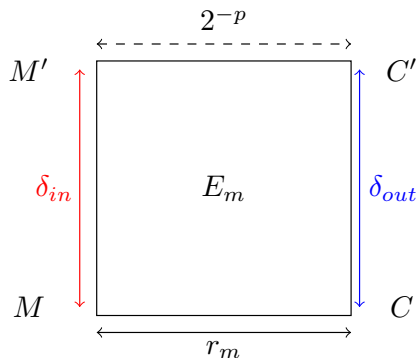
- **Time complexity**: number of computations to succeed $\rightsquigarrow < 2^k$ encryptions.
- **Data complexity**: number of queries used $\rightsquigarrow < 2^n$.
- **Memory complexity**: size of memory used.

\rightsquigarrow Many possible Trade-offs.

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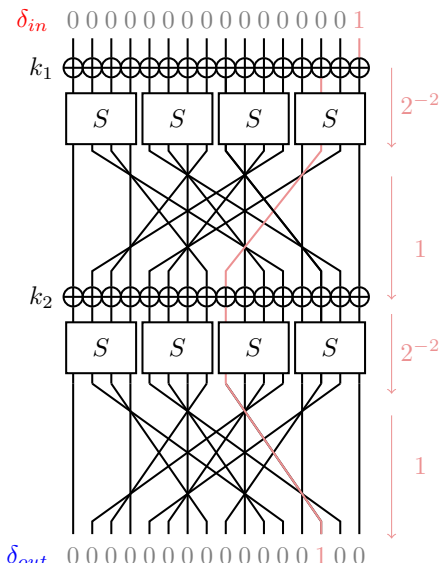
Differential Cryptanalysis

Introduced to the public by Biham and Shamir in 1990 in [BS90].



- Distinguishes if $2^{-p} \gg 2^{-n}$.
- $(\delta_{in}, \delta_{out})$ is called a differential.

An example

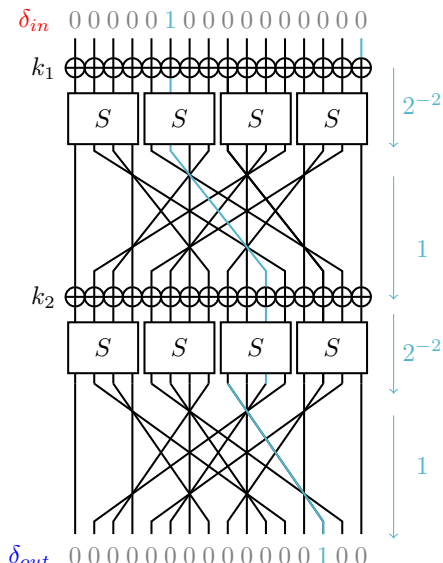


- Probability of the path
 $2^{-4} \gg 2^{16}$.

- Difference distribution table:

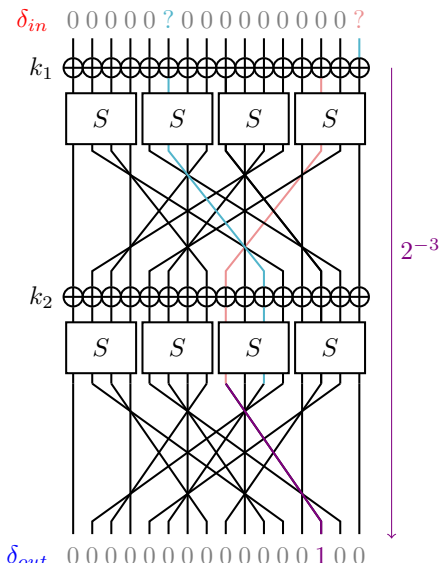
	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	4	4	4	4	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	4	0	0	4	2	2	2	2	2
3	0	0	0	0	0	0	0	4	0	0	4	2	2	2	2	2
4	0	4	0	0	0	4	0	0	2	2	0	2	0	0	2	2
5	0	4	0	0	4	0	0	0	2	2	0	2	0	0	2	2
6	0	4	0	0	4	0	0	0	2	2	0	0	2	2	0	2
7	0	4	0	0	0	0	4	0	2	2	0	0	2	2	0	2
8	0	0	4	4	0	0	0	4	0	4	0	0	0	0	0	0
9	0	0	0	0	2	2	2	2	0	0	0	0	2	2	2	2
a	0	0	0	0	2	2	2	2	4	0	4	0	0	0	0	0
b	0	0	4	4	0	0	0	0	0	0	0	0	2	2	2	2
c	0	0	2	2	2	2	0	0	0	2	0	2	2	0	2	0
d	0	0	2	2	0	0	2	2	0	2	0	2	0	2	0	2
e	0	0	2	2	0	0	2	2	0	2	0	2	0	2	0	2
f	0	0	2	2	2	2	0	0	0	2	0	2	0	2	0	2


An example



- Probability of the path $2^{-4} \gg 2^{-16}$.
- Same output as other path.

An example

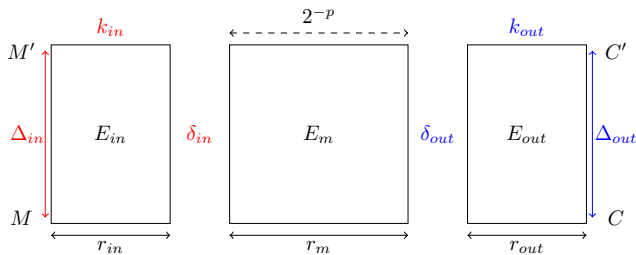


- Probability of the path $2^{-4} \gg 2^{16}$.
- Same output as other path.
- Combine both differential characteristic \rightsquigarrow probability increases.
-  This probability might be wrong (quasi-differential...).

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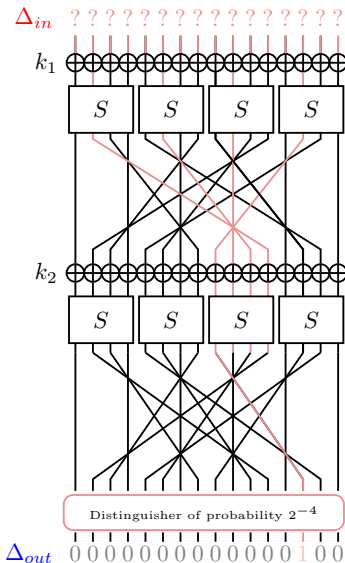
Key-recovery attack

Let $E = E_{out} \circ E_m \circ E_{in}$ be a block cipher.



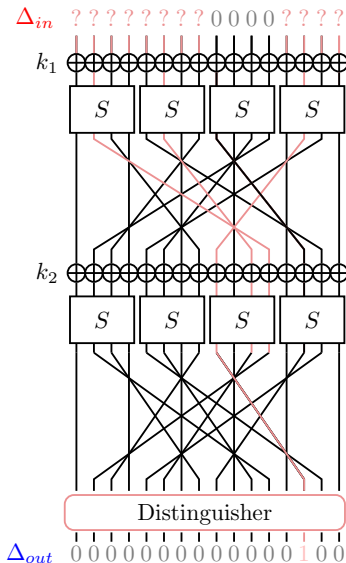
- Δ_{in} and Δ_{out} are the sets of differences that can lead to δ_{in} and δ_{out} .
- Find candidate triplets $(P, P', k_{in} \cup k_{out})$ that imply δ_{in} and δ_{out} .
- The time complexity depends in part on the size of k_{in} and k_{out} .

An example



- Propagate the differences δ_{in} and δ_{out} .
- Build "structure" of plaintexts, the space of pairs of plaintext taking all the possible values for the ? bits.
- Filter when their corresponding ciphertexts using Δ_{out} .
- Guess the all the bits of k_1 and 4 bits of k_2 .

Improving key-recovery attacks



- Study the possible weaknesses of the Sbox.
- Other existing techniques to improve key-recovery attacks \rightsquigarrow State-test technique: guess the bit of the state instead of the key.
- Recover information on the key through **non-linear** equations.

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Thank you for your attention !